A direct approach to fault-tolerance in measurement-based quantum computation via teleportation

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Abstract. We discuss a simple variant of the one-way quantum computing model [R. Raussendorf and H.-J. Briegel, PRL 86, 5188, 2001], called the Pauli measurement model, where measurements are restricted to be along the eigenbases of the Pauli X and Y operators, while qubits can be initially prepared both in the $|+_{\frac{\pi}{4}}\rangle := 1/\sqrt{2}(|0\rangle + e^{i\frac{\pi}{4}}|1\rangle)$ state and the usual $|+\rangle := 1/\sqrt{2}(|0\rangle + |1\rangle)$ state. We prove the universality of this quantum computation model, and establish a standardization procedure which permits all entanglement and state preparation to be performed at the beginning of computation. This leads us to develop a direct approach to fault-tolerance by simple transformations of the entanglement graph and preparation operations, while error correction is performed naturally via syndrome-extracting teleportations.

1. Introduction

The one-way quantum computation (1WQC) model [1] has been widely studied since its discovery. One particular issue that has attracted attention is how to perform fault-tolerant (FT) quantum computation (QC) in such a model [2, 3, 4, 5, 6, 7]. While FTQC can be performed through such a model by simulating FT quantum circuits via 1WQC [4, 5], FTQC can also be achieve directly through the use of topological error correction techniques [6, 7]. The focus of this paper is to illustrate another direct approach to FTQC in measurement based computation, building on insights into the measurement calculus [8] and generalizations of 1WQC, as well as teleportation-based approaches to error correction [9, 10].

We consider a model where measurements in the full XY-plane are traded off for more complex preparations of the vertices in the entangled resource state. This model, which we call the *Pauli measurement model* (PMM), uses only measurements along the X and the Y directions, while the entangled resource state is obtained via initialization of individual qubits into the state $|+\rangle := \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ or $|+_{\frac{\pi}{4}}\rangle := \frac{1}{\sqrt{2}}(|0\rangle + e^{i\frac{\pi}{4}}|1\rangle)$, followed by application of the unitary interaction $\wedge Z := \mathrm{diag}(1, 1, 1, -1)$ (also known as the controlled-Z gate) between certain pairs of qubits. We show that the PMM model is fault-tolerant in the usual simulation sense [4, 5]. Moreover, through the use of encoded or nested graph states [11], and the careful selection of quantum codes, all necessary operations for computation can be performed transversally on encoded information, so that the graph state computation itself is made fault-tolerant if the error rate is low enough.

First, we investigate how to extend the main properties of the 1WQC model using these modified preparation states, while still maintaining the properties one needs for convenient error correction. We then demonstrate that this model naturally provides the resources necessary for fault-tolerant syndrome extraction, and illustrate how any PMM computation can be transformed into a larger one that has a lower effective error rate if the error rate per operation is below some threshold, achieving fault-tolerance.

2. One-way quantum computation with phase preparation

Consider a slight extenstion to 1WQC where a measurement pattern, or simply a pattern, is defined by a sequence of quantum operations over a finite set of qubits V, along with two subsets $I \subseteq V$ and $O \subseteq V$ representing the pattern inputs and outputs respectively (I and O may intersect). The allowed operations are: (a) N_i^α, preparation of qubit i in the state $|+_\alpha\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\alpha}|1\rangle)$; (b) E_{ij}, unitary interaction between qubits i, j of the form $\wedge Z$; (c) M_i^α, measurement of qubit $i \notin O$ in the $|\pm_\alpha\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm e^{i\alpha}|1\rangle)$ eigenbasis, with outcome $s_i \in \{0, 1\}$ corresponding to collapse into the state $|+_\alpha\rangle$ or $|-_\alpha\rangle$, respectively; (d) X_j and $Z_j(\alpha) := e^{-i\frac{\alpha}{2}Z_j}$, local unitaries on qubit j. In addition, local unitaries and measurement basis may depend on the outcome of measurements of other qubits, which is denoted in the natural way, e.g. $X_j^{s_k}$ indicating a unitary which acts if $s_k = 1$, or $M_j^{\alpha - s_k\beta}$ indicating a measurement in a basis which depends on the measurement outcome s_k.

Measurements are considered to be destructive, and we require that no operations be performed on measured qubits. We also only consider runnable patterns where no operations depend on the outcome of measurements that have not yet been performed. Local unitaries are crucial for the understanding of how universality and determinism

come about (recall that measurement outcomes in quantum mechanics are, in general, non-deterministic) [3, 8]. Both 1WQC and PMM are particular cases of this more general model: to obtain the 1WQC model set $\alpha = 0$ in clause (a); to obtain the PMM, set $\alpha = 0, \pi/4$ in clause (a), and $\alpha = n\pi/2$ in clauses (c) and (d).

Patterns, denoted by gothic letters, e.g. $\mathfrak{A}$ and $\mathfrak{B}$, can be combined to create a new pattern via parallel concatenation $\mathfrak{A}\|\mathfrak{B}$, or serial concatenation $\mathfrak{A} \circ \mathfrak{B}$. Parallel concatenation means the qubits are relabelled in such a way that all operations in $\mathfrak{A}$ commute with all the operations in $\mathfrak{B}$ – if $\mathfrak{A}$ implements the unitary U_A, and $\mathfrak{B}$ implements U_B, then $\mathfrak{A}\|\mathfrak{B}$ implements $U_A \otimes U_B$. Serial concatenation means the output of $\mathfrak{A}$ is fed into the input of $\mathfrak{B}$ – that is, $\mathfrak{A} \circ \mathfrak{B}$ implements the unitary $U_B U_A$.

As an example, consider the pattern

$$\mathfrak{J}_\alpha := X_2^{s_1} M_1^{-\alpha} E_{12} N_2^0, \tag{1}$$

with $(V, I, O) = (\{1, 2\}, \{1\}, \{2\})$. Given an arbitrary state ρ on qubit 1, this sequence of operations implements $J_\alpha := HZ(\alpha)$ on the input state and places the resulting state $J_\alpha \rho J_\alpha^\dagger$ on qubit 2. This is one of the fundamental building blocks for 1WQC [8], since it allows for arbitrary one qubit rotations. Any of the local unitaries considered can be merged with a (destructive) measurement as follows:

$$M_i^\alpha Z_i(\beta) = M_i^{\alpha-\beta} \tag{2}$$

$$M_i^\alpha X_i \quad = M_i^{-\alpha} \tag{3}$$

and it is readily seen that the $\mathfrak{J}_\alpha$ pattern above is the serial concatenation of a $Z(\alpha)$ rotation with a modified one-bit teleportation (implementing H) – a well known result for 1WQC [1, 12, 13]. Patterns which lie outside 1WQC model can also be expressed in this extended model, such as

$$\mathfrak{X}_\alpha := X_3^{s_2} Z_3^{s_1} M_2^{-(-1)^{s_1}\alpha + \frac{\pi}{4}} M_1^0 E_{23} E_{12} N_2^{\frac{\pi}{4}} N_3^0 \tag{4}$$

with $(V, I, O) = (\{1, 2, 3\}, \{1\}, \{3\})$, which implements the unitary $HZ(\alpha)H = e^{-i\frac{\alpha}{2}X} = J_\alpha J_0$. It follows from the equations above, that this pattern is equivalent to a $Z(\alpha)$ conjugated by a one-qubit teleportation. The importance of writing the pattern in this form, using the $N_2^{\frac{\pi}{4}}$ preparation, becomes clear when measurements are restricted to the X or Y eigenbasis, as will be discussed later.

Other patterns which play an important role are $\wedge\mathfrak{Z}$, $\mathfrak{N}$ and $\mathfrak{M}$, defined as follows: $\wedge\mathfrak{Z} := E_{12}$, with $(V, I, O) = (\{1, 2\}, \{1, 2\}, \{1, 2\})$, implements the unitary $\wedge Z$; $\mathfrak{N} := N_i^0$ implements initialization of qubit i into the state $|+\rangle$; and, $\mathfrak{M} := M_i^0$ implements the measurement of qubit i in the $|\pm\rangle$ X eigenbasis. These patterns are crucial in order to fulfill the DiVincenzo criteria [14].

The usual protocol for 1WQC requires computation to be performed in three steps: individual qubit state preparation, entangling operations between qubits, and measurement of individual qubits with feed-forward of outcomes. In order to follow this protocol for the generalized model, patterns must be put into a *standard form* where any computation can be performed by a sequence of operations in this order. Note that these steps do not include the application of single qubit unitaries, but adaptive measurements can be used to address this absence, since all quantum computations must end with the measurement of the qubits in order for information to be extracted. Once a pattern is in standard form, it is convenient to consider the entangled state that is prepared for the computation. Such a state can be described by an *entanglement graph*, with vertices V and edges (i, j) for every command E_{ij} in

the pattern, where the vertices are labelled with the initial state in which the qubit is prepared.

The process of turning a given pattern into a pattern in standard form is called *standardization*. The rewrite rules needed for this procedure are simply (2) and (3), along with conjugation relation between unitaries, $E_{12}X_1 = X_1 Z_2 E_{12}$, and $E_{12}Z_1(\alpha) = Z_1(\alpha)E_{12}$, as well as all the free commutation relations between operations on different qubits. Simple rewriting theory arguments [8] show that by applying the conjugation relations to move all the local unitaries towards the left in the pattern, and then by applying (2) and (3), any pattern can be put in standard form.

As mentioned previously, PMM is obtained by setting (i) state preparation angles to 0 or $\frac{\pi}{4}$, (ii) measurement angles to $\frac{n\pi}{2}$, and (iii) local unitaries to X and $Z(\frac{n\pi}{2})$. Two simple facts follow from this: first, PMM is closed under standardization and concatenation, as can be readily seen from the merging and conjugation relations above; second, PMM contains the patterns $\wedge\mathfrak{Z}$, $\mathfrak{J}_\alpha$, $\mathfrak{X}_\beta$, $\mathfrak{N}$ and $\mathfrak{M}$, where $\alpha = \frac{n\pi}{2}$ and $\beta = \frac{n\pi}{2} + \frac{\pi}{4}$, as well as their concatenations. In particular, $\mathfrak{X}_{\frac{\pi}{4}}$ allows for an operation outside the Clifford group while requiring only Pauli measurements.

Corollary. *The PMM is approximately universal in the Solovay-Kitaev sense.*

This construction of a universal gate set is equivalent to the construction of fault-tolerant universal gate sets via teleportation [12, 15].

3. Fault-tolerance

3.1. Simulation approach

In reality, physical implementations of any computational model are susceptible to noise. In principle, such physical implementation can be made fault-tolerant by encoding the data and the operations in a manner such that, even after the overhead of such an encoding is considered, one can efficiently perform computations of arbitrary size [2, 16, 17, 18, 19]. The noise model that is usually considered, and which we restrict ourselves to in this work, is the model of independent random failure of each of the operations during computation. One approach to achieve fault-tolerance in 1WQC is by using fault-tolerance in the circuit model as a stepping stone. The construction of fault-tolerant circuits is well understood [12, 15], and it is now well known that the implementation of such circuits via 1WQC can lead to fault-tolerant quantum computation [5, 4]. This can be most simply understood and demonstrated through the idea of composable simulations [5, 20], and the same idea carries through to the PMM with minor modifications. The main distinction is that in the PMM, the change of measurement bases dependent on measurement outcomes corresponds to a local Clifford correction, as opposed to a local Pauli correction. Thus the noisy simulations through the PMM will have an error model which consists of random application of local Clifford operators. However, because of the linearity of quantum mechanics and the fact that the Pauli group forms a basis for all single qubit operators, the errors are still correctable as in simulations through the 1WQC model. Thus, simulating fault-tolerant quantum circuits through the PMM model is also fault-tolerant.

3.2. Intrinsic fault-tolerance

We now turn our attention to the possibility of making any PMM computation directly fault-tolerant, instead of simulating fault-tolerant quantum circuits within 1WQC.

1WQC relies on frequent measurement to implement a desired state evolution, but none of this information is used towards fault-tolerance in simulation-based approaches. The opportunity for improved performance becomes evident once one considers the well known link between teleportation and 1WQC [13, 20], and the fact that FTQC in the circuit model can achieve very high thresholds via extensive use of teleportation for simultaneous syndrome extraction and state evolution [9].

3.2.1. Encoded Computation Before we consider how syndrome information is to be extracted, we must consider encoded computation in the PMM. The basic elements of the PMM are: preparation of qubits in either $|+\rangle$ or $|+_{\frac{\pi}{4}}\rangle$, pair-wise entanglement via $\wedge Z$, and measurement in the X or Y eigenbases depending on the outcomes of previous measurements. Given some quantum code, we can consider these same elements, but in the subspace corresponding to the code chosen – that is, preparation of a block of qubits in the encoded states above, encoded entangling operations, and collective measurements in the encoded eigenbases X and Y. The use of the 7 qubit self-orthogonal doubly-even CSS codes [21] simplifies the problem considerably if the generators of the encoded Pauli operators are chosen to be $\overline{Z} = Z^{\otimes 7}$ and $\overline{X} = X^{\otimes 7}$. In that case, the encoded entangling operation $\overline{\wedge Z}$ is given by the transversal application of $\wedge Z$ gates between respective qubits in two blocks – in the PMM, it is the parallel concatenation of the pattern $\wedge\mathfrak{Z}$. Moreover, measurement in the encoded X and encoded Y eigenbases are performed by measuring each of the qubits within the code block in the same basis individually, followed by classical decoding of the outcomes to determine the encoded outcome. If we consider concatenated encoding using this 7 qubit code, i.e. $\overline{X}^{(j)} = \left(\overline{X}^{(j-1)}\right)^{\otimes 7}$ for the jth level of encoding with $\overline{X}^{(0)} \equiv X$ and similar relations for $\overline{Z}^{(j)}$, these transversality properties are preserved.

The encoding procedure of any given stabilizer code over qubits is known to correspond to a pattern in 1WQC which allows for arbitrary input and requires only measurements along the eigenbases of the Pauli operators X and Y [22, 23] – this includes both the isomorphism between stabilizer codes and graph codes, as well as the necessary local Clifford corrections. If we restrict the inputs to be either $|+\rangle$ or $|+_{\frac{\pi}{4}}\rangle$, we can obtain the encoded states $|\overline{+}\rangle$ or $|\overline{+}_{\frac{\pi}{4}}\rangle$ strictly within the PMM. The entanglement graph corresponding to the encoding circuit for the 7 qubit code is depicted in Figure 1. Concatenated encoding proceeds in the obvious way, by serial concatenation of the measurement pattern corresponding to the encoding procedure.

However, for the purpose of FTQC, encoding requires verification of the encoded states in order to ensure that these state do not contain errors that are too correlated [2, 24]. This can be performed naturally in the PMM via state encoding at some given level of concatenation, followed by syndrome extracting teleportation of the lower levels of encoding [9]. There are purification protocols for the entangled state corresponding to the encoding procedure of any CSS code [25], – such as the 7 qubit code, as depicted in Figure 1 – which may also be employed to reduce errors and error correlations. We consider only the encoded states that have been successfully verified after preparation as part of the computation. In this manner, encoded computation in the PMM is akin to computation with nested graph states [11], where the entanglement graph for encoding is nested within the computation entanglement graph.

It is important to note that the entire concatenated graph state must not be purified directly, since the maximum vertex degree of the resulting graph grows linearly with the level of concatenation, and the purification protocol performance degrades

with higher vertex degrees [25]. In order to avoid this problem, one may perform purification per level of concatenation separately, followed by syndrome extraction teleportation with post-selection of the states which have a clean syndrome.

Previous proposals for fault-tolerance in the 1WQC model make use of what is called the *one-buffered implementation* of cluster states [4]. In such implementations, which are based on the simulation of quantum circuits, the entanglement subgraph corresponding to the first two time steps in the circuit model is prepared. The measurements corresponding to the first time step are performed, followed by the state preparation and entagling operations corresponding to the third time step of the circuit model. After that, the measurements for the second time step are performed, and computation proceeds keeping a one time step "buffer" of qubits, so that the entire entanglement graph need not be prepared in one shot. However, it has been demonstrated that the 1WQC model, as well as the PMM, allow for greater parallelism in the computation [8]. In particular, some sequences of operations which lie in multiple time steps in the circuit model can be performed in a single time step in these measurement models (a large class of such operations are unitaries in the Clifford group). Thus, one may prepare states corresponding to larger subgraphs of the entanglement graph where all non-output qubits will be measured simultaneously. There is a partial order constraint for the timing of the measurements which is implied in the definition of the PMM (as well as the 1WQC model), and this partial order gives the dependencies between the measurements [8]. One can therefore associate a subgraph of the entanglement graph with each time step where a collection of measurements may be performed in parallel. In the case of the PMM, measurement of a vertex prepared in the $|+_{\frac{\pi}{4}}\rangle$ introduces a local Clifford correction to qubits connected to it in the entanglement graph, and thus such vertices will always be on the boundary of the subgraphs. However, patterns implementing Clifford operations have measurements which are independent of eachother's outcome, and thus the insertion of Clifford operations in a pattern does not increase the number of such subgraphs, or equivalently, the minimal number of time steps in which measurements can be performed in parallel. This is particularly relevant for fault-tolerance, as encoding and syndrome extraction operations for stabilizer codes are Clifford operations. In principle such operations can be performed in the same time step, if the entire corresponding subgraph is available for measurement. The preparation of the subgraph itself will require multiple time step, due to verification, error correction and purification at different levels of encoding, but since these operations are independent of the rest of the computation, they may be performed offline. Clearly, it is not required that maximal parallelism – corresponding to the largest subgraph – be implemented. There is a trade-off between the overhead introduced by more complex offline preparation and verification of such larger subgraphs, and the lower effective error rate which may be achieved. Implementations may range from the one-buffered approach, to the fully parallel approach, which ensures that all measurements without dependencies can be performed simultaneously.

3.2.2. Syndrome extraction In order to perform FTQC, one must be able to extract information about the errors in the data in order to ensure that only clean enough states are introduced into the computation, as described in the previous section, but also to obtain information about which errors are likely to have occurred in order to correct them. This error syndrome extraction can be performed via teleportation, as recently described in [9, 10]. In essence, the idea is to start with a maximally

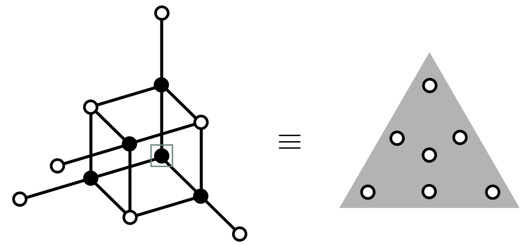

Figure 1. Entanglement graph corresponding to the encoding of a single qubit into the 7 qubit CSS code. The boxed node corresponds to an arbitrary input qubit. All but the white qubits (corresponding to the encoding output) are measured in the X basis (up to feed-forward-based corrections).

entangled pair of encoded qubits $|\overline{\Omega}\rangle_{1,2} = \overline{\wedge Z}_{12}|\overline{+}\rangle_1|\overline{+}\rangle_2$ which is prepared offline. Given some encoded state $\overline{\rho}$, the error syndrome can be extracted in the following manner. Measure each transversal pair of physical qubits from $\overline{\rho}$ and the first half of $|\overline{\Omega}\rangle_{1,2}$ in a basis of maximally entangled states. The state $\overline{\rho}$ is then teleported into the second half of the entangled pair, up to a tensor product g of local Pauli operators which is inferred from the outcomes of the pair measurements. The error syndrome can in turn be inferred from these corrections by considering the commutator of g with each of the generators of the stabilizer group of the code. This protocol can be seen as the transversal teleportation of all the physical qubits where the n maximally entangled pairs have been projected into the codespace being used. Note that this is different from an encoded teleportation – an encoded maximally entangled state is used, but the measurements are performed on physical qubits, not encoded qubits. This proposed technique for FTQC has not been rigorously proven to have an error threshold as is the case for many other techniques [2], but extensive numerical evidence supports such a claim [9].

Although the usual teleportation protocol [26] is performed with Bell pairs and measurement in the Bell basis, teleportation can be performed with any measurement in a basis of maximally entangled states, and this choice of basis fixes which maximally entangled states can be used as a resource. In fact, teleportation can be performed by the serial concatenation $\mathfrak{J}_0 \circ \mathfrak{J}_0 = X_3^{s_2} Z_3^{s_1} M_2^0 M_1^0 E_{23} E_{12} N_3^0 N_2^0$, which may be understood as a teleportation using the basis obtained by applying a Hadamard gate to one of the qubits of a Bell basis. If we allow for modified preparation of the entangled resource state, the pattern, stripped of the entanglement preparation, simply becomes $\mathfrak{T} = X_3^{s_2} Z_3^{s_1} M_2^0 M_1^0 E_{12}$, which, for completeness, must be concatenated with the pattern for the modified entangled state preparation (i.e. the pattern that prepares the encoded entangled state).

Thus, in the PMM, syndrome extraction of some encoded state $\overline{\rho}$ is performed by: (I) preparing and verifying the encoded state $|\overline{\Omega}\rangle_{12}$, (II) teleporting all qubits in $\overline{\rho}$ individually using the resource state $|\overline{\Omega}\rangle_{12}$, and (III) performing classical post-processing to infer the syndrome information from the teleportation measurement outcomes. As discussed, step (I) can be performed by hierarchical teleportation and post-selection [9, 10]. Step (II) can be performed by parallel concatenation of the pattern $\mathfrak{T}$ above, while step (III) is merely classical post-processing which affects the bases of subsequent measurements. Partial syndrome information can be extracted in a similar fashion, as in the case of the $\mathfrak{J}_\alpha$ pattern with $\alpha = \frac{n\pi}{2}$, where, depending on

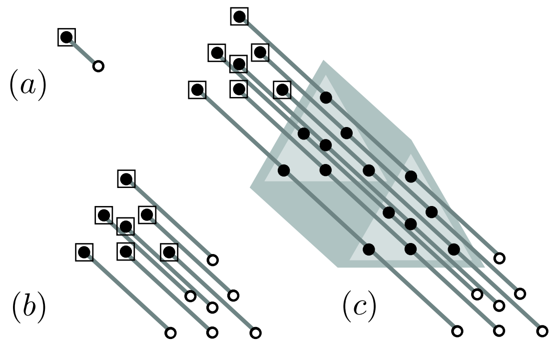

Figure 2. Entanglement graphs for the fault-tolerant implementation of $\mathfrak{J}_0$. The boxed nodes correspond to input qubits, and all but the white nodes (corresponding to output qubits) are measured in the X eigenbasis (up to feed-forward-based corrections).

α, one can obtain information about Pauli errors which anti-commute with X or Y.

3.2.3. Performing the computation Given any measurement pattern in the PMM, one may make it fault-tolerant by first translating each of the commands with a larger pattern representing its encoded form, then inserting instances of the syndrome extracting teleportation between each operation, and standardizing the resulting pattern.

As a simple example, consider the pattern fragment $X_2^{s_1} M_1^0 E_{12}$ that implements the unitary $J_0 = H$, with entanglement graph depicted by Figure 2(a). Using a single level of encoding under the 7 qubit CSS code, the resulting pattern is already long and omitted for brevity, but its entanglement graph in Figure 2(b) demonstrates the simplicity of the transformation. The subgraph enclosed in the shaded triangle corresponds to the encoded state that must be prepared and verified before the remaining operations can be performed, in what can be seen as an extension of the one-buffered implementation of the unencoded case [4]. With the data protected by an error correction code and offline preparation of encoded qubits, one inserts the syndrome extracting teleportation to obtain the final fault-tolerant pattern with corresponding entanglement graph depicted in Figure 2(c). Again, the subgraph inside the irregular pentagon (corresponding to the preparation of the encoded maximally entangled pair) is to be prepared and verified before the qubits within it interact with the remainder of the graph. This demonstrates the fact that only three subgraphs need to be prepared and verified offline: the smaller subgraphs corresponding to the encoded states $|\overline{+}\rangle$ and $|\overline{+}_{\frac{\pi}{4}}\rangle$, and the larger subgraph corresponding to the encoded state $|\overline{\Omega}\rangle$. This procedure for implementing fault-tolerance works for any linear graph. Other graphs, such as the one corresponding to a $\wedge Z$ pattern interacting between two linear chains, can be handled in a similar fashion, by simply inserting syndrome extracting teleportations before and after the $\wedge Z$ pattern.

It is important to note that the qubits, interactions and measurements added to the computation in order to extract syndrome information correspond to Clifford operations on the quantum states. As pointed out earlier in the paper, the measurements associated with a sequence of Clifford operations can be performed

in any order, even simultaneously and immediatelly after the qubits are made available for measurement, and thus they do not increase the depth complexity of the computation [3, 8]. Moreover, this also allows for the offline preparation of subgraphs corresponding to Clifford operations, along with measurement of parts of the subgraph, which allows for the elimination of some types of error via post-selection – as pointed out in [27], for the case of repeated syndrome extraction, one can post-select on subgraphs which will yield agreeing syndromes.

4. Conclusion

We have described a measurement based model of computation with the notable feature that measurements are restricted to the eigenbases of the Pauli operators X and Y, and qubit state preparation is extended to both $|+\rangle$ and $|+_{\frac{\pi}{4}}\rangle$. With the appropriate choice of quantum codes, any measurement pattern in this model can be directly modified into another pattern within the same model, which, according to numerical evidence [9], will have a lower effective error rate as long as the failure rate per operation is below a threshold.

After the completion of this work we became aware of similar work by Fujii and Yamamoto [28], where numerical simulations indicate that the error threshold is comparable with the one obtained in [9].

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